The last two sections taught rules that differentiate certain classes of functions. What if those rules do not cover a certain function? In this section, we will learn how to combine those rules to differentiate any function of a differentiable function using the *Chain Rule*.

# The Chain Rule

Some functions are not of but of another function, that one being of . For example: is a function (square root) of another function (addition) of the number one and another function (the square of ). The chart below illustrates the functions in that :

In the past, each differentiation rule (from sections 2-3 and 2-4) would cover only one function, or one chevron in the diagram above. The chain rule links them together.

The chain rule uses a variable to extract the insides of a function.

For any function where , the derivative is:

|  |
| --- |
|  |
|  |
|  |

The chain rule works from **outside to inside**. In the chevron chart above, differentiate the square root first, which will naturally require differentiating the function before it, and so on, up to , in which case the chain rule will no longer apply.

# Proof of the Chain Rule

Let’s find of .

Suppose is differentiable at .

Let .

Let .

Let’s define that for any derivative, epsilon () is the difference between the difference quotient and derivative, and is zero for a continuous function.

Using algebra, that equation can be rearranged to:

Let’s apply this combining the derivatives of and .

Substitute ,

Limiting and and ,

Substituting ,

# Power Rule Combined with the Chain Rule

Suppose .

Let .

# How Would You Answer?

* What is the chain rule?
* Why is the chain rule true?
* How can the chain rule differentiate the power of a function?